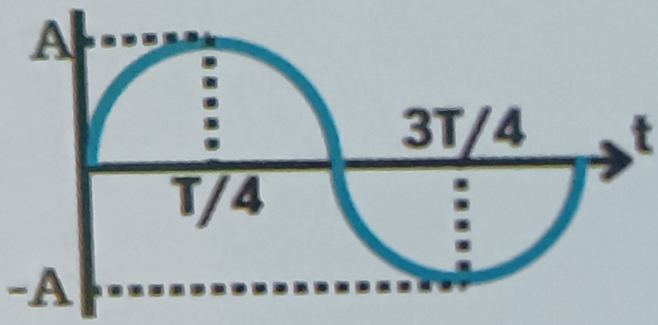
Oscillations

Angular Frequency
T = Time period
f = frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$

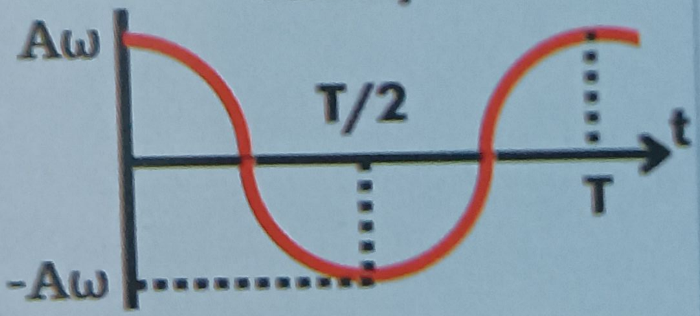
Displacement



$$y = A\sin(\omega t + \phi)$$

A = amplitude
ϕ = initial phase
(at mean position $\phi = 0$)

Velocity

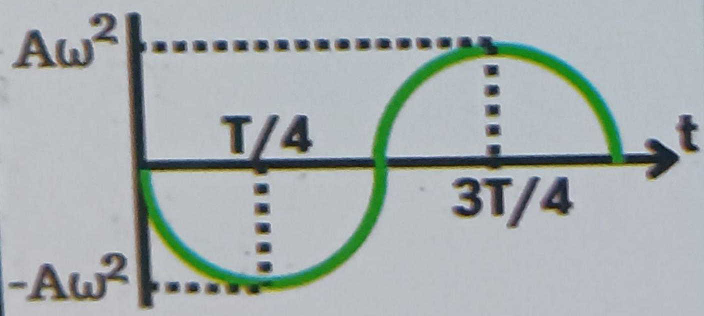


$$v = A\omega\cos(\omega t + \phi)$$

$$v = \omega\sqrt{A^2 - y^2}$$

(in terms of position)

Acceleration



$$a = -A\omega^2\sin(\omega t + \phi)$$

$$a = -\omega^2 y$$

Time t	0	$T/4$	$2T/4$	$3T/4$
Displacement, x	0 (min)	A (max)	0 (min)	-A (max)
Velocity, v	$A\omega$ (max)	0 (min)	$-A\omega$ (max)	0 (min)
Acceleration, a	0 (min)	$-A\omega^2$ (max)	0 (min)	$A\omega^2$ (max)

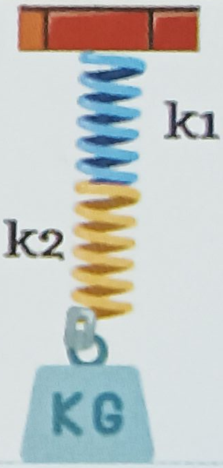
Kinetic Energy	$K = \frac{1}{2}m\omega^2(a^2 - x^2) = \frac{1}{2}k(a^2 - x^2)$
Potential Energy	$\frac{1}{2}kx^2$
Total Mechanical Energy	$\frac{1}{2}ka^2$ (constant)

Pendulum

Simple Pendulum g_{eff} = g, when whole system is at rest $g_{eff} = \sqrt{a^2 + g^2}$, in horizontal acceleration g_{eff} = g-a, in deaccelerating reference frame g_{eff} = g+a, in accelerating reference frame	$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$
Compound Pendulum/Physical Pendulum	$T = 2\pi\sqrt{\frac{I}{mgd}}$
Spring Pendulum m = mass of suspended object k = force constant	$T = 2\pi\sqrt{\frac{m}{k}}$

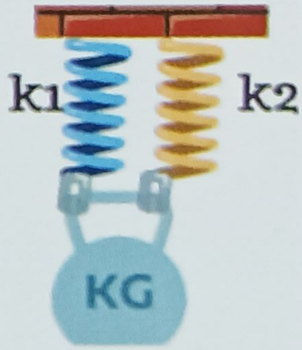
Series Combination of Spring

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



Parallel Combination of Spring

$$k = k_1 + k_2$$



Damped Oscillation

Damping Force

$$\vec{F} = -b\vec{v}$$

Equation of Motion

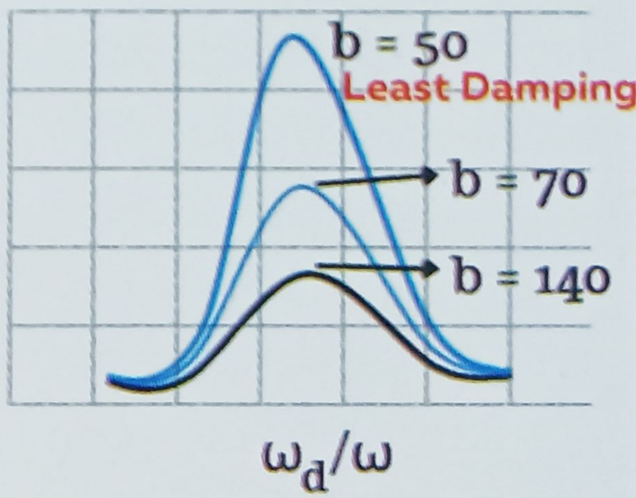
$$\frac{mdv}{dt} = -kx - bv$$

$$\omega' = \sqrt{\left(\frac{k}{m}\right) + \left(\frac{b}{2m}\right)^2}$$

Forced Oscillation and Resonance

Small Damping

$$A = \frac{F_0}{m(\omega^2 - \omega_d^2)}$$



Driving Frequency Close to Natural Frequency

$$A = \frac{F_0}{w_d b}$$

- The x - t graph of a particle performing simple harmonic motion is shown in the figure. The acceleration of the particle at t=2 s is : $-\pi^2/16$